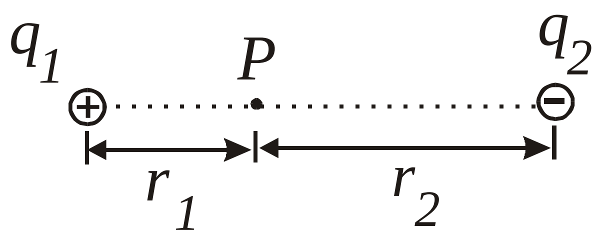
AP Physics – More Electric Fields - 6

We have learned how to calculate the force that acts on a test charge that is near two or more charges. We can also find the potential difference. But what about the electric field?



What is the electric field at point *P* between the two charges?

The voltage at *P* from the charges is: $V = \frac{1}{4\pi \in_0} \frac{q}{r}$

The electric field is related to the potential difference by: $E = \frac{V}{d}$

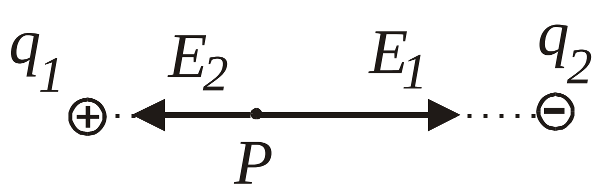
Let's solve this equation for the potential difference:

$E = \frac{V}{d}$ $\quad V = Ed$ $\quad$ the distance ***d*** is simply ***r***, so:

$V = Er$ $\quad$ We can plug this into the equation for potential difference above:

$V = \frac{1}{4\pi \in_0} \frac{q}{r}$ $\quad Er = \frac{1}{4\pi \in_0} \frac{q}{r}$ $\quad E = \frac{1}{4\pi \in_0} \frac{q}{r^2}$

We can draw the two electric field vectors:



Let's put some numbers to the thing. Let q_1 = 22.5 μC and q_2 = -35.5 μC. r_1 = 22.0 cm and r_2 = 42.5 cm. Find the electric field at ***P***.

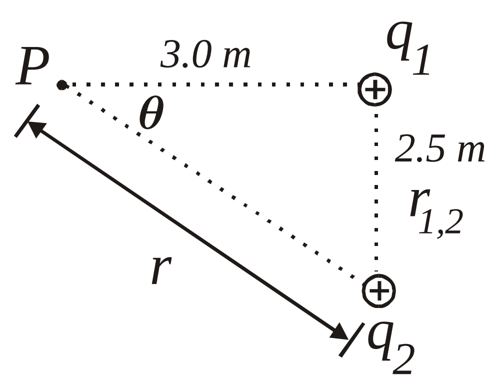
$$E_1 = \frac{1}{4\pi \in_0}\frac{q_1}{r_1^2} \quad = \left(8.99\,x\,10^9\,\frac{Nm^2}{C^2}\right)\left(\frac{22.5\,x\,10^{-6}\,C}{(0.22\,m)^2}\right) \quad = 4179\,x\,10^6\,\frac{N}{C}$$

$$E_2 = \frac{1}{4\pi \in_0}\frac{q_1}{r_1^2} \quad = \left(8.99\,x\,10^9\,\frac{Nm^2}{C^2}\right)\left(\frac{-33.5\,x\,10^{-6}\,C}{(0.425\,m)^2}\right) \quad = -1667\,x\,10^6\,\frac{N}{C}$$

Since the two vectors are in opposite directions, we can just add them:

$$E = E_1 + E_2 \quad = 4179\,x\,10^3\,\frac{N}{C} - 1667\,x\,10^3\,\frac{N}{C} \quad = \boxed{2510\,\frac{N}{C}}$$

- Two charges are situated near point P. The angle θ is 33°. q_1 = 0.025 μC and q_2 = 0.12 μC. Find (a) the potential difference at ***P***? (b) The electric field strength at point ***P***?



(a) We can find the voltage (potential difference) at ***P*** from each of the two charges, then add them up.

$$V = \frac{1}{4\pi \in_0}\frac{q}{r} \qquad V_1 = \frac{1}{4\pi \in_0}\frac{q_1}{r_1}$$

$$V_1 = \frac{1}{4\pi \in_0}\frac{q_1}{r_1} = \left(8.99\,x\,10^9\,\frac{Nm^2}{C^2}\right)\left(0.025\,x\,10^{-6}\,C\right)\left(\frac{1}{3.0\,m}\right) \quad = 0.0749\,x\,10^3 V \quad = 74.9\,V$$

We need to find the distance from ***P*** to q_1.

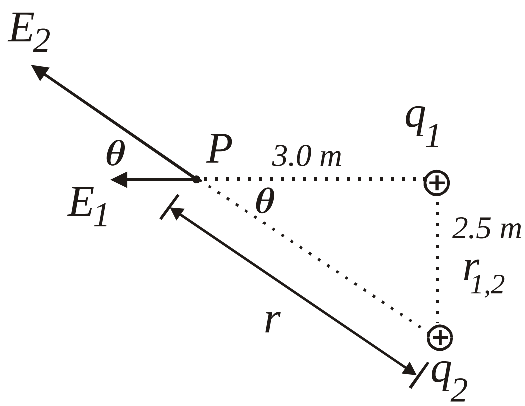
$$r = \sqrt{3.0^2 + 2.5^2} = 3.9m$$

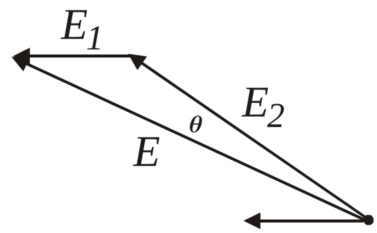
$$V_2 = \left(8.99\,x\,10^9\,\frac{Nm^2}{C^2}\right)\left(0.12\,x\,10^{-6}\,C\right)\left(\frac{1}{3.9\,m}\right) \quad = 277V$$

$$V = V_1 + V_2 \quad = 74.9\,V + 277\,V \quad = \boxed{352\,V}$$

(b) Finding the field is easy too. First we find the field strength produced by each charge at point P. These will be vectors, so after we find them we have to add them up using vector addition.

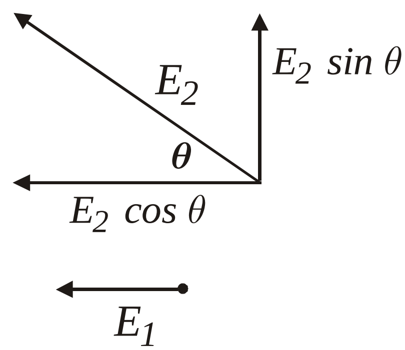
The two electric field vectors look something like this:





$$E = \frac{V}{d} \qquad E_1 = \frac{V_1}{d_1} = \frac{74.9\,V}{3.0\,m} = 25.0\,\frac{V}{m}$$

$$E_2 = \frac{277\,V}{3.9\,m} = 71\,\frac{V}{m}$$



Now we can add the vectors. We need to look at the old ***x*** and ***y*** component deal.

We'll have up and to the left as the positive direction.

$$\sum E_x = E_1 + E_2 \cos\theta$$

$$\sum E_y = E_2 \sin\theta$$

For ***y***: $E_y = E_2 \sin\theta = \left(71\,\frac{V}{m}\right)\sin 39.8^{\circ} = 45.4\,\frac{V}{m}$

For ***x***: $E_1 + E_2\cos\theta = 25.0\,\frac{V}{m} + 54.5\,\frac{V}{m} = 79.5\,\frac{V}{m}$

Now we can find the magnitude of the electric field strength:

$$E = \sqrt{E_y^{\,2} + E_x^{\,2}} = \sqrt{\left(45.4\,\frac{V}{m}\right)^2 + \left(79.5\,\frac{V}{m}\right)^2} = \boxed{91.6\,\frac{V}{m}}$$

For the direction of the vector, we can use the tangent function with the ***x*** and ***y*** components.

$$\tan\theta = \frac{E_y}{E_x} = \frac{45.4\,\frac{V}{m}}{79.5\,\frac{V}{m}} \qquad \boxed{\theta = 29.7^{\circ}}$$