AP Physics – Potential Difference - 5

By convention, a point in an electric circuit is said to have zero electric potential (potential difference) if it is grounded (connected to earth).

The other way to have zero potential difference is to have a point charge be located at an infinite distance from another charge.

We've been looking at a test charge placed in an electric field; we learned how to determine the potential difference on the test charge, the change in potential energy of charge, and the work needed to move the charge within the field.

We can look at the field itself and calculate the potential difference at a point within the field using this equation:

$$V = \frac{1}{4\pi \in_0} \frac{q}{r}$$

V is the potential difference at a point that is a distance ***r*** from the charge ***q*** that created the field.

The Potential difference depends only on charge and the distance from the charge.

Superposition Principle: When there are two or more charges in proximity to one another, the change in potential energy is the sum of the potential difference for each charge. Potential difference is a scalar, therefore:

> ***Total electric potential at some point near several point charges is the algebraic sum of the electric potentials from each of the charges***

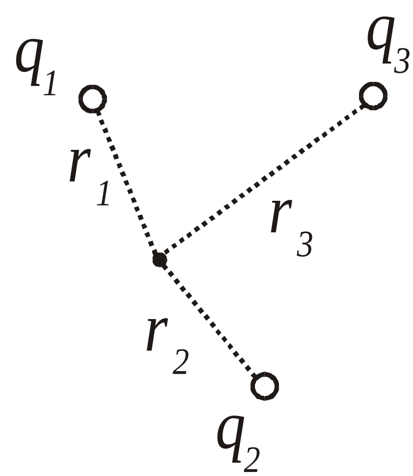
This can be written in general form as:

$$V = \frac{1}{4\pi \in_0} \sum_i \frac{q_i}{r_i}$$

This shows that for a number of charges, the voltage is the algebraic sum of $\frac{1}{4\pi \in_0} \frac{q}{r}$ for each charge.

This is the equation you will be provided on the AP Physics Test.

Three charges are arranged as shown in the drawing to the right. To find the potential difference at the point in the center, you calculate the potential difference for q_1, q_2, and q_3. You can do this if you know the value for the charge and the distance to the central point. Lastly, to find the total potential difference, you simply add up the three ***V'***s that you found for the three points.



Let's put some numbers into the mix. Let $q_1 = 1.5\ \mu C$, $q_2 = 2.0\ \mu C$, and $q_3 = 2.5\ \mu C$. For the distances, r_1 = 2.0 cm, r_2 = 1.8 cm, and r_3 = 2.7 cm.

$$V = \frac{1}{4\pi \in_0} \sum_i \frac{q_i}{r_i}$$

$$V_1 = \frac{1}{4\pi \in_0} \frac{q_1}{r_1} = \left(8.99\,x\,10^9 \frac{Nm^2}{C^2}\right)\left(1.5\,x\,10^{-6} C\right)\left(\frac{1}{2.0\,x\,10^{-2} m}\right) = 6.7\,x\,10^5\,V$$

$$V_2 = \left(8.99\,x\,10^9 \frac{Nm^2}{C^2}\right)\left(2.0\,x\,10^{-6} C\right)\left(\frac{1}{1.8\,x\,10^{-2} m}\right) = 10\,x\,10^5\,V$$

$$V_3 = \left(8.99\,x\,10^9 \frac{Nm^2}{C^2}\right)\left(2.5\,x\,10^{-6} C\right)\left(\frac{1}{2.7\,x\,10^{-2} m}\right) = 8.3\,x\,10^5\,V$$

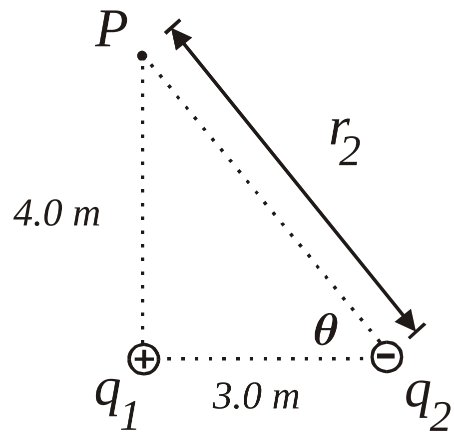
Now, having found each of the potential differences for the charges, we can find the total potential difference.

$$V = V_1 + V_2 + V_3$$

$$V = 6.7\,x\,10^5\,V + 10\,x\,10^5\,V + 8.3\,x\,10^5\,V = 25\,x\,10^5\,V = \boxed{2.5\,x\,10^6\,V}$$

An important thing to understand is that potential difference is not a vector, it's a scalar, so you add them up algebraically.

- Two charges are situated as shown. The distances are given in the drawing. The charges have the following values: q_1 is 5.0 μC and q_2 is -2.0 μC. (a) Find the potential difference at point ***P***. (b) How much work is required to bring a third point charge of 4.0 μC from infinity to ***P***?



(a) Find the potential difference at ***P***.

$$V = \frac{1}{4\pi \in_0} \frac{q}{r}$$ for q_1

$$V_1 = \left(8.99\,x\,10^9 \frac{Nm^2}{C^2}\right)\left(5.0\,x\,10^{-6} C\right)\left(\frac{1}{4.0\,m}\right)$$

$$V_1 = 11.2\,x\,10^3 V \quad = 1.12\,x\,10^4 V$$

We must now find r_2:

$$r_2 = \sqrt{(4.0\,m)^2 + (3.0\,m)^2} = 5.0\,m$$

$$V_2 = \left(8.99\,x\,10^9 \frac{Nm^2}{C^2}\right)\left(-2.0\,x\,10^{-6} C\right)\left(\frac{1}{5.0\,m}\right) \quad = -0.360\,x\,10^4 V$$

$$\Delta V = \Delta V_1 + \Delta V_2$$

$$\Delta V = \left(1.12\,x\,10^4 V\right) + \left(-0.360\,x\,10^4 V\right) \quad = 0.760\,x\,10^4 V \quad = \quad \boxed{7.6\,x\,10^3\,V}$$

(b) How much work to bring a third point charge of 4.0 μC from infinity to P?

The work is equal to the potential energy change of the move.

$$U = qV \quad W = qV \quad = \left(4.0\,x\,10^{-6} C\right)\left(7.6\,x\,10^3 \frac{J}{C}\right)$$

$$W = 30.4\,x\,10^{-3} J \quad = \quad \boxed{3.0\,x\,10^{-2} J}$$

Velocity and Electric Fields: We can use all this impressive new knowledge about potential difference, potential energy, and work to solve impressive problems. In fact, you will be required to solve such problems to be successful on the AP Physics Test.

- Calculate (a) the energy of a proton that is accelerated from rest through a potential difference of 120 V and (b) the speed of an electron accelerated from rest through a potential difference of 120 V.

Potential energy that the proton loses will be equal to the kinetic energy it will have after it has been accelerated through the field.

$$U = qV = W \qquad W = K = \frac{1}{2}mv^2 \qquad so \qquad \frac{1}{2}mv^2 = qV \qquad v = \sqrt{\frac{2qV}{m}}$$

$$v = \sqrt{\frac{2qV}{m}} = \sqrt{\frac{2\left(1.60\, x\, 10^{-19} \cancel{C}\right)\left(120\, \frac{J}{\cancel{C}}\right)}{1.67\, x\, 10^{-27}\, kg}}$$

$$v = \sqrt{229.9\, x\, 10^8\, \frac{m^2}{s^2}} = 15\, x\, 10^4\, \frac{m}{s} = \boxed{1.5\, x\, 10^5\, \frac{m}{s}}$$

- Through what potential difference would an electron need to accelerate to achieve a velocity of 1.80 x 10^7 m/s?

We employ the same reasoning as in the previous problem: the work is the same as the potential energy that goes into the system. We can then set that equal to the kinetic energy that the electron ends up with after its been accelerated through the field.

$$U = qV = W \qquad \frac{1}{2}mv^2 = qV \qquad \text{Solve for potential difference}$$

$$V = \frac{mv^2}{2q} = \frac{\left(9.11\, x\, 10^{-31} kg\right)\left(1.80\, x\, 10^7\, \frac{m}{s}\right)^2}{2\left(1.60\, x\, 10^{-19} C\right)} = \boxed{9.22\, x\, 10^2\, V}$$

- An electron is 3.00 cm from the center of a uniformly charged sphere of radius 2.00 cm. The sphere's charge is 1.00 x 10 $^{-9}$ C. How fast will the electron be traveling when it hits the surface of the sphere?

Conservation of Energy:

$$U_i = K_f$$

$$\frac{k_e q_1 q_2}{r} = \frac{1}{2}mv^2$$

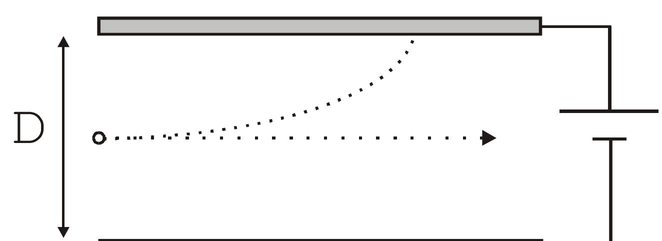
$$v = \sqrt{\frac{2k_e q_1 q_2}{rm}}$$

$$v=\sqrt{\frac{2\left(8.99\,x\,10^{9}\,\frac{\cancel{kg}\cdot\cancel{m}\cdot m^{2}}{s^{2}\cdot\cancel{C}^{2}}\right)\left(1.60\,x\,10^{-19}\,\cancel{C}\right)\left(1.00\,x\,10^{-9}\,\cancel{C}\right)}{\left(0.0300\,\cancel{m}-0.0200\,\cancel{m}\right)\left(9.11\,x\,10^{-31}\,\cancel{kg}\right)}}$$

$$v=\sqrt{315.8\,x\,10^{12}\,\frac{m^{2}}{s^{2}}}=17.8\,x\,10^{6}\,\frac{m}{s}=\boxed{1.78\,x\,10^{7}\,\frac{m}{s}}$$

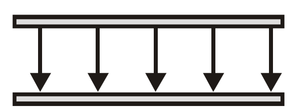
- An electron is fired into at the midpoint of a field between two charged plates. The initial velocity of the electron is 5.6 x 10^{6} m/s. The plates are 2.00 mm apart. The $\boldsymbol{\Delta V}$ for the plates is 100.0 V. (a) Find the magnitude of the electric field. (b) Make a drawing of the two plates and put in some arrows showing the direction of the field. (c) Determine where the electron will hit on the upper plate.

(a) We need to find the magnitude of the field. We can use the equation for the electric field to find the field. We're ignoring the minus sign as we're only interested in the magnitude of the field.



$$E=\frac{V}{d}=\frac{100\,V}{0.0020\,m}=5.00\,x\,10^{4}\,\frac{V}{m}$$

(b) From the drawing, we see that the field is oriented so that the force on the electron is upward. This means that the field must point downwards. Recall that the direction of the field is the direction that a positive test charge would be deflected. A negative charge would go in the opposite direction.



(c) An upward force is exerted on the electron as it travels through the field. We can find the force from the field:

$$E=\frac{F}{q}\qquad F=Eq\quad=\left(5.00\,x\,10^{4}\,\frac{N}{\cancel{C}}\right)\left(1.60\,x\,10^{-19}\,\cancel{C}\right)=8.00\,x\,10^{-15}\,N$$

Since we know the force acting on the electron, we can use the second law to find the acceleration.

$$F=ma\qquad a=\frac{F}{m}$$

$$a=\frac{8.00\,x\,10^{-15}\,\frac{\cancel{kg}\cdot m}{s^{2}}}{9.11\,x\,10^{-31}\,\cancel{kg}}=0.878\,x\,10^{16}\,\frac{m}{s^{2}}=8.78\,x\,10^{15}\,\frac{m}{s^{2}}$$

Note -- acceleration of gravity is insignificant compared with the acceleration form the field, so we can ignore it.

Next we find the time it takes for the electron to hit the upper plate. Since it's in the center of the plates when it enters, it will be accelerated a vertical distance of 1 mm.

$$y = \frac{1}{2}at^2 \qquad \text{y is 1.00 mm or } 10^{-3} \text{ m}$$

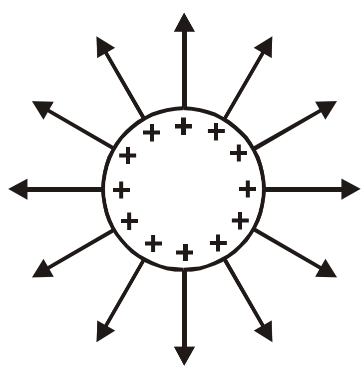
$$t = \sqrt{\frac{2y}{a}} = \sqrt{\frac{2\left(1.00 \, x \, 10^{-3} \, m\right)}{8.78 \, x \, 10^{15} \, \frac{m}{s^2}}} = \sqrt{0.2278 \, x \, 10^{-18} \, s^2} = 0.477 \, x \, 10^{-9} \, s$$

We know the time till the electron hits, we also know its initial horizontal velocity, and we know that it will travel horizontally at a constant speed. So knowing the time and the speed, we can find the horizontal distance it travels before the electron hits.

$$x = v_x t = 5.6 \, x \, 10^6 \, \frac{m}{s}\left(0.477 \, x \, 10^{-9} \, s\right) = 2.7 \, x \, 10^{-3} \, m = \boxed{2.7 \quad mm}$$

Equipotential Curves:

Equipotential curves can be drawn for a charge. For example if we look at a positively charged conducting sphere, the lines of force would look like this:



Now if we draw a curve around the sphere that represents places that have the same potential difference, we get a sphere. It will be a circle on our two dimensional drawing. This is because the voltage only depends on the distance from the charge. The charge is, of course, evenly distributed on the sphere. The voltage drops off with distance, so the curves nearest the surface of the sphere would have the greatest potential.

The equation for potential difference is simply: $V = \frac{1}{4\pi \epsilon_0} \sum_i \frac{q_i}{r_i}$

Which simplifies to: $$V = \left(\frac{1}{4\pi \in_0}\right)\frac{q}{r}$$

As the distance (r) increases, the potential difference decreases. So if we draw equipotential curves that represent a specific voltage difference – say each curve represents a decrease of 5 volts, it would look like this: