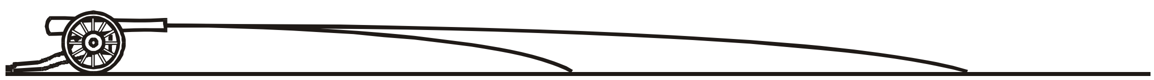
AP Physics – Satellite Stuff

One of Sir Isaac Newton's greatest claims to fame is his explanation of how the planets orbit the sun. That and the ability to compute the orbits, orbital speeds, orbital periods, &tc. Before Newton, the motion of the heavens was a mystery. After Newton, the motion of the heavens was an explainable physical phenomenon.

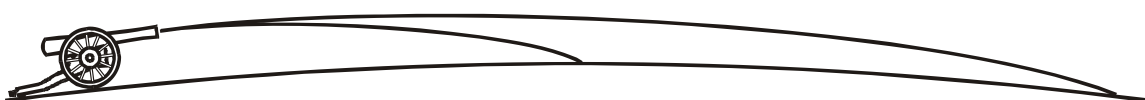
Let's do us a "mind experiment". This is an experiment where you think instead of do. Anyway, picture a cannon that is set to fire horizontally. What does the path of the projectile look like?



Path of short range projectile

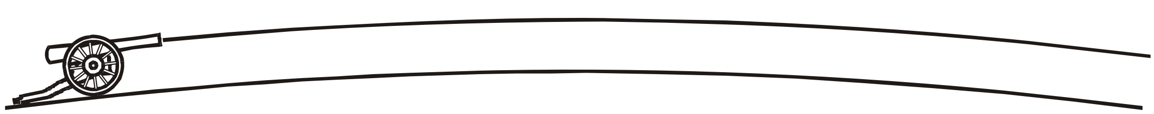
The projectile will follow a curved path. This is because it is being accelerated downwards by the force of gravity. The greater the velocity of the projectile, the farther it will go before it strikes the Earth.

The Earth, however, is not flat, although over short distances we can pretend that it is. So what actually is happening is that the projectile moves over and falls to the ground on a curved surface. So we have a curved path and a curved surface. We have to take the curvature of the Earth into account when firing long range projectiles. Possible paths would look like this:



Path of long range projectile

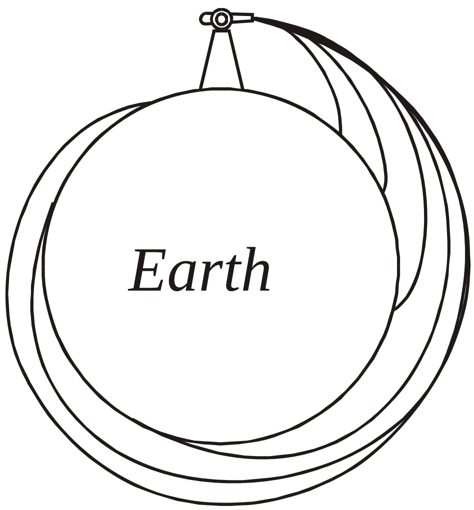
Again, the greater the velocity of the projectile, the greater the range. Newton showed that if the velocity was great enough, the curving path of the falling projectile would match the curved surface of the earth and the falling projectile would never actually hit the Earth. Here is a drawing of this.



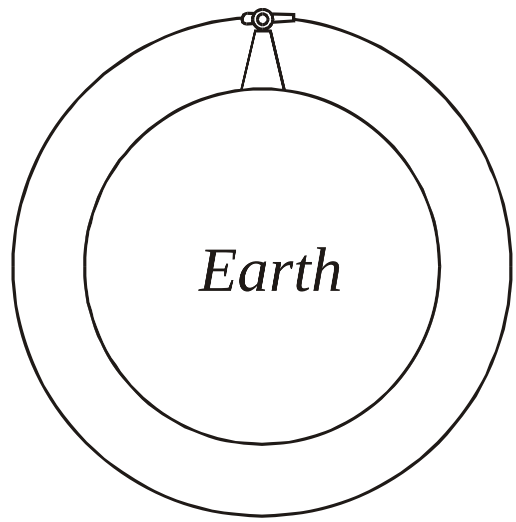
Path of projectile with same curvature as surface of the earth

Newton imagined a mountain on the earth that was so high that its summit was outside of the earth's atmosphere (this eliminates friction with air). On top of the mountain is a powerful cannon. The cannon fires a projectile horizontally. The projectile follows a curved path and eventually hits the earth. Now we add more gunpowder to the charge and fire another cannonball. This cannonball will travel a greater distance before it too hit the surface of the earth. We keep firing the gun with a bigger and bigger charge. The cannonball goes further and further before it strikes the earth.

Eventually the velocity is great enough so that the curved path of the projectile matches the curved surface of the earth and the cannonball never gets closer to the planet's surface. It keeps falling forever.



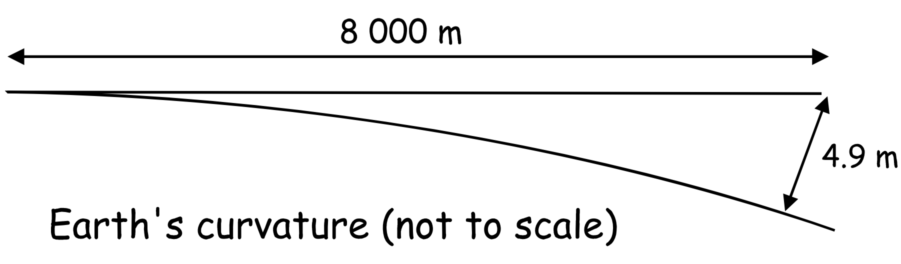
Path of projectile fired with larger and larger charges



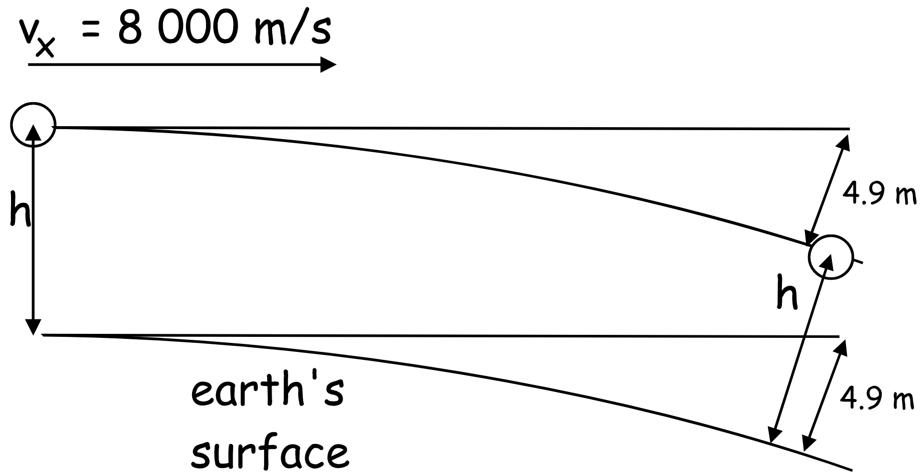
With just the right velocity, the projectile never reaches the earth's surface

That basically is your orbit.

The earth is not flat. It is a sphere and its surface has a fairly constant curvature. The surface drops 4.9 m in 8 000 m of horizontal travel.



If we launch a cannonball with a velocity of 8 000 m/s, it will fall a distance of 4.9 m and travel a horizontal distance of 8 000 m in one second. This means that it will stay at the same height above the earth's surface throughout its path. Of course, if we did this near the surface, we'd have the air slowing the projectile down. We'd also have to worry about the cannonball running into houses and mountains and trees and so forth. Above the atmosphere, however, all these impediments are eliminated.



The orbit of the everyday celestial object is described by a combination of the law of gravity, Newton's three laws, and the stuff we just learned about, circular motion.

> Sherlock Holmes is the most portrayed character on film, having been played by 72 actors in 204 films. The historical character most represented in films is Napoleon Bonaparte, with 194 film portrayals. Abraham Lincoln is the U.S. President to be portrayed most on film, with 136 films featuring actors playing the role.

Orbital Equations: Let us assume that the orbit of a satellite about the earth (or any other massive body) is a circle. Most orbits are not circles but are instead ellipses. This was discovered by Johanes Keppler in the 1600's. But let's keep it simple and look at a circular orbit.

In order to have a circular path, a centripetal force is required. This is supplied by the force of gravity between the two bodies.

So we can set the centripetal force equal to Newton's law of gravity:

$$F = G\frac{m_1 m_2}{r^2} \qquad \textbf{\textit{gravity}} \qquad F_C = \frac{m_2 v^2}{r} \qquad \textbf{\textit{centripetal force}}$$

Set them equal to one another:

$$G\frac{m_1 m_2}{r^2} = \frac{m_2 v^2}{r}$$

Notice how the mass of the object canceled out.

$$v^2 = \frac{Gm_1}{r} \qquad v = \sqrt{\frac{Gm_1}{r}}$$

This gives us an equation for the orbital velocity:

$$v = \sqrt{\frac{Gm}{r}}$$

The mass, ***m***, in the equation is the mass of the body being orbited. If we are talking about a planet orbiting the sun, then the mass we would use would be that of the sun. The mass of the satellite cancels out, so its mass is not involved in the orbital velocity equation at all.

The equation for the orbital velocity will not be given you on the AP Physics Test. So be prepared to derive it if you need it.

Period of satellite: This is another simple derivation job. The period of a satellite is ***T***, the time to make one orbit. What would be the period of the earth around the sun?

Let's develop the equation for the period of a satellite. We'll use the equation for distance and solve it for the time:

$$v = \frac{x}{t} \qquad t = \frac{x}{v}$$

d, the distance traveled is the circumference of the orbit. We know that it would be:

$$x = 2\pi r$$

So we can plug that in to the equation we solved for time:

$$t = \frac{2\pi r}{v}$$

but v is also given by the equation we just derived for the orbital velocity:

$$v = \sqrt{\frac{Gm}{r}}$$

If we plug the orbital velocity into our working equation, i.e., put them together, we get:

$$t = \frac{2\pi r}{\sqrt{\frac{Gm}{r}}}$$

Square both sides: $t^2 = \dfrac{4\pi^2 r^2}{\dfrac{Gm}{r}}$

Clean up everything up nice and neatlike using our potent algebra skills:

$$t^2 = 4\pi^2 r^2 \frac{r}{Gm} \qquad t^2 = \frac{4\pi^2 r^3}{Gm}$$

$$t = \sqrt{\frac{4\pi^2 r^3}{Gm}} \qquad t = 2\pi\sqrt{\frac{r^3}{Gm}}$$

And we end up with an equation for the period of a satellite. Again the mass in the thing is the mass of the body being orbited:

$$t = 2\pi\sqrt{\frac{r^3}{Gm}}$$

Now let's do some exciting problems.

- What is orbital velocity of the earth around the sun? The sun has a mass of 1.99×10^{30} kg, the mean distance from the earth to the sun is 1.50×10^{11} m.

$$v = \sqrt{\frac{Gm}{r}} = \sqrt{6.67 \, x \, 10^{-11} \frac{kg \cdot m \cdot m^2}{s^2 \cdot kg^2}\left(1.99 \, x \, 10^{30} \, kg\right)\frac{1}{1.50 \, x \, 10^{11} \, m}}$$

$$v = \sqrt{8.85 \, x \, 10^8 \frac{m^2}{s^2}} = \boxed{2.97 \, x \, 10^4 \frac{m}{s}}$$

- A satellite is in a low earth orbit, some 250 km above the earth's surface. r_{earth} is 6.37×10^6 m and $m_{earth} = 5.98 \times 10^{24}$ kg. Find the period of the satellite in minutes.

r is the radius of the earth plus ***y***, the height of the satellite. $r = r_{earth} + y$

$$y = 250 \, km\left(\frac{10^3 \, m}{1 \, km}\right) = 0.25 \, x \, 10^6 m$$

$$r = 6.37 \text{ x } 10^6 \text{ m} + 0.25 \text{ x } 10^6 \text{m} = 6.62 \text{ x } 10^6 \text{ m}$$

$$T = 2\pi\sqrt{\frac{r^3}{Gm}} \quad = 2\pi\sqrt{\frac{\left(6.62\ x\,10^6\,m\right)^3}{6.67\ x\,10^{-11}\frac{kg\ m}{s^2}\frac{m^2}{kg^2}\left(5.98\ x\,10^{24}\,kg\right)}}$$

$$T = 2\pi\sqrt{\frac{290.12\ x\,10^{18}}{39.89\ x\,10^{13}}s^2} \quad = 2\pi\sqrt{7.273\ x\,10^5 s^2}$$

$$T = 2\pi\sqrt{72.73\ x\,10^4 s^2} \quad = 53.57\ x\,10^2\ s \quad = \quad 5357\ s$$

$$T = 5357\ s\left(\frac{1\,\text{min}}{60\ s}\right) \quad = \quad \boxed{89.3\,\text{min}}$$

- The moon has a period of 28 days. If the earth's mass is 5.98 x 10^{24} kg, how far is the moon from the earth?

$$T = 28\ day\left(\frac{24\ h}{1\ day}\right)\left(\frac{3600\ s}{1\ h}\right) \qquad T = 2.42\ x\,10^6\ s$$

$$T = 2\pi\sqrt{\frac{r^3}{Gm}} \qquad \text{square both sides:} \qquad T^2 = 4\pi^2\left(\frac{r^3}{Gm}\right)$$

Solve for r^3:

$$r^3 = T^2 Gm\left(\frac{1}{4\pi^2}\right) \quad r = \sqrt[3]{T^2 Gm\left(\frac{1}{4\pi^2}\right)}$$

$$r = \sqrt[3]{\left(2.42\,10^6\,s\right)^2 6.67\ x\,10^{-11}\frac{kg\ m}{s^2}\frac{m^2}{kg^2}\left(5.98\ x\,10^{24}\,kg\right)\left(\frac{1}{4\pi^2}\right)}$$

$$r = \sqrt[3]{5.923\ x\,10^{25} m^3}$$

convert 5.923 x 10^{25} to something times 10^{24} (we chose 10^{24} because 24 is divisible by 3)

$$r = \sqrt[3]{59.23\ x\,10^{24} m^3} \quad = \quad \boxed{3.90\ x\,10^8\,m}$$

Useless Tivia Facts:

- 15 million blood cells are destroyed in the human body every second.
- 3,000 teens start smoking every day in the United States.
- The average person spends two weeks of their life kissing.
- Scientists in Australia's Parkes Observatory thought they had positive proof of alien life, when they began picking up radio-waves from space. However, after investigation, the radio emissions were traced to a microwave in the building.
- 40 people are sent to the hospital for dog bites every minute.
- In Japan, 20% of all publications sold are comic books.
- The average day is actually 23 hours, 56 minutes and 4.09 seconds. We have a leap year every four years to make up for this shortfall.
- In the next seven days, 800 Americans will be injured by their jewelry.
- Disneyland opened in 1955.
- It is estimated that at any one time, 0.7% of the world's population are drunk.
- If you divide the Great Pyramid's perimeter by two times it's height, you get pi to the fifteenth digit.
- The pupil of the eye expands as much as 45 percent when a person looks at something pleasing.
- 50,000 of the cells in your body will die and be replaced with new cells all while you have been reading this sentence.
- 67.5% of men wear briefs instead of boxers.
- 75% of people wash from top to bottom in the shower.
- The average four year-old child asks over four hundred questions a day.
- 80% of all body heat escapes through the head.
- Lightning strikes the Empire States Building more than 50 times a year.
- 85% of the population can curl their tongue into a tube.
- A fetus acquires fingerprints at the age of three months.
- The Future's Museum in Sweden features a scale model of the solar system. The sun is 105 meters in diameter and the planets range from 5mm to 6 km from the ‘sun’. The exhibit also features the nearest star, Proxima Centauri. The distance to the star is also to scale. It is located in the Museum of Victoria. . . . in Australia.

Gravity in Orbit: We all know that the astronauts in space orbiting the earth are "weightless". Does this mean that there is no gravity in space? Well, no. Most of our spacecraft are in pretty low orbits. The distance between the astronauts and the earth is not that much greater than when they are on the earth. Their weight is only about ten percent less than it is on earth. So why are they weightless?

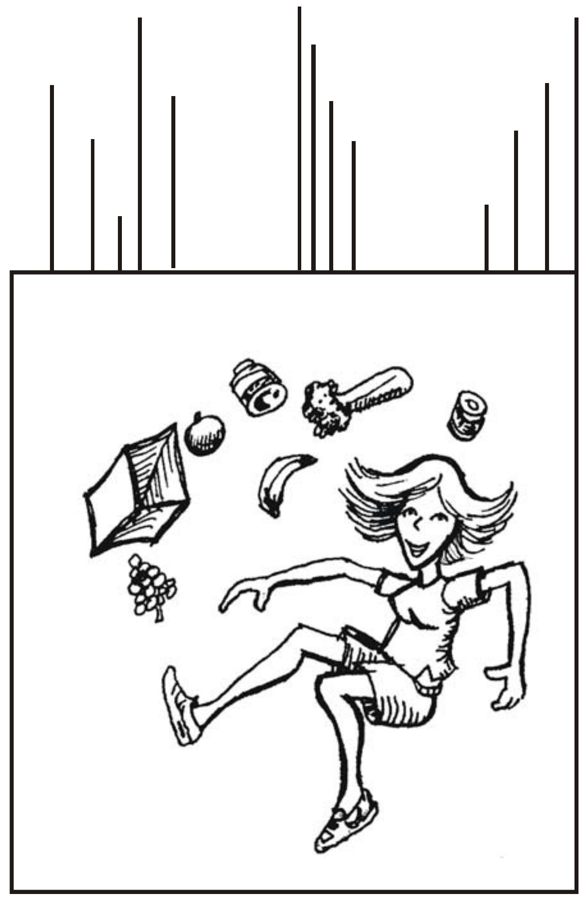
The space shuttle and everything in it are falling towards the earth. It is in a state of freefall. We know that everything falls at the same rate, so everything in the space shuttle is falling at the same speed. Because of this there is no relative motion between the space shuttle and everything in it. There is no sense of up or down and the astronauts no longer feel the force of gravity. It’s like being inside an elevator that is falling down the elevator shaft (the cable broke or something). In a normal elevator, one that isn’t falling, gravity exerts a downward force on everything. If you stand on a bathroom weight scale, you push down on it and it reads out your weight. But now the cable breaks. You are still standing on the scale, but the elevator, the scale, and you are all falling down accelerating at 9.8 meters per second squared. You no longer exert a force on the scale – it is falling at the same speed that you are. It now reads zero.

Any objects in the elevator would appear to be weightless. If you held a ball outward and then released it, it would not appear to fall down (since it is already falling). It would appear to float in space in front of you. You would think, "Hey, cool, there's no gravity in the elevator. Neat!"

It would be pretty neat too. Until the elevator hits the bottom of the elevator shaft.

Okay, let's transfer this idea to the space shuttle. It is, in effect, a falling elevator, one with the advantage of not crashing into the floor – it never hits the earth!

That's why the astronauts are "weightless".



Free Falling in an elevator

Dear Cecil:
Tycho Brahe, the astronomer, is buried here in Prague in a church. That's not so unusual. What is rather strange is the persistent rumor that he had a silver nose, something to do with a duel. Did it tie on around his face with silk strings? Did it have little hooks that went around his ears, like eyeglasses? Was it surgically attached? I'm truly curious. I also wonder if it tarnished, and whether he had to polish it. Did it make a note like a whistle when he blew his nose? --Raymond Johnston, chief copy editor, *Prague Post*

Cecil replies:
Glad to be of help, Ray. I know how tough it is for journalists to come by this kind of information. What you heard was no rumor. Tycho Brahe (1546-1601), the father (or at least the godfather) of modern astronomy, really did wear an artificial nose, owing to the fact that the real one had been sliced off in a duel. You may think: this does not sound like the scientists I know. Tycho Brahe, however, was no ordinary stargazer.

It happened in 1566 while the 20-year-old Tycho was studying at the University of Rostock in Germany. Attending a dance at a professor's house, he got into a quarrel with one Manderup Parsbjerg, like himself a member of the Danish gentry. Over a woman? Nah--tradition has it that the two were fighting over some fine point of mathematics. (My guess: Fermat's Next-to-Last Theorem, which posits that 2 + 2 = 5 for very large values of 2.) Friends separated them, but they got into it again at a Christmas party a couple weeks later and decided to take it outside in the form of a duel. Unfortunately for Tycho the duel was conducted in pitch darkness with swords. Parsbjerg, a little quicker off the dime, succeeded in slicing off the bridge (apparently) of Tycho's nose. Reconstructive surgery then being in a primitive state, Tycho concealed the damage as best he could with an artificial bridge made of precious metals. He carried some nose goop with him always, either to polish the nose or to glue it more firmly in place. But no hooks or string, and probably no whistling either.

High-handed and irascible, Tycho Brahe was the kind of guy who got into duels. Luckily he was also a genius. Fascinated by the stars since his youth, he discovered that existing astronomical tables were grossly inaccurate and set about making his own meticulous observations of the heavens, a project that occupied him for most of his life. To keep him from going abroad, the king of Denmark and Norway gave Tycho a prodigious quantity of cash ($5 billion in today's money, by one estimate) and his own island. There Tycho constructed an observatory where for 20 years he compiled the impressive body of astronomical data that his assistant Johannes Kepler subsequently used to deduce the laws of planetary motion. All this, mind you, with the naked eye; the telescope hadn't yet been invented.

To give you a further indication of the type of guy we're dealing with there, Tycho didn't marry the mother of his eight children, employed a dwarf as a jester, kept a pet elk (which died after breaking a leg while going downstairs drunk), dabbled in alchemy, and tyrannized the local peasantry. After his royal patron died of excessive drink he managed to tick off everyone in Denmark, had his subsidies revoked, and eventually found it wise to leave the country. Having relocated to Prague, he died after drinking heavily at dinner, obviously a pretty common fate in those days.

Tycho's tomb was reopened in 1901 and his remains were examined by medical experts. The nasal opening of the skull was rimmed with green, a sign of exposure to copper. Presumably this came from the artificial nose, which supposedly had been made of silver or gold. The experts put the best face on this, as it were, saying that Tycho was an expert in metallurgy and probably wanted an alloy that was durable and skin colored. Sure, guys. I say Mr. Astronomy got nicked in the nose department twice.

--CECIL ADAMS

Dear Doctor Science,

IF YOUR SO SMART TELL ME HOW THE INERTIAL DAMPENERS WORK ON THE USS ENTERPRISE-NCC-1701-D? BET I STUMPED YOU ON THAT ONE.

-- JON STIGLER from EADS, TN

Dr. Science responds:

They didn't work, and that was the big problem with the series. The ship slipped into hyperdrive whenever it felt like it, sending the crew zipping across the galaxy in a nanosecond. This made it especially hard on the navigators, who were constantly coming up with novel ways to get their bearings, including monitoring talk radio from nearby planets in the hope of learning their approximate location. Eventually, captain and crew developed a pervasive sense of chronic apathy, and instead of piloting their craft, spent their time watching reruns of the first Star Trek, where Jim and Spock traded profundities on the planet of scantily clad women.