Homework 6

These questions are intended to give you practice in using the gravitational law. They will give you a feeling for typical forces with a range of masses and also how sensitive force is to distance.

Useful data

$G = 6.67 \times 10^{-11}$ N m^2 kg^{-2}

Earth's mass = 5.97×10^{24} kg

Moon's mass = 7.34×10^{22} kg

Sun's mass is 2.0×10^{30} kg

Radius of the Moon = 1.64×10^{6} m

Radius of the Earth = 6.37×10^{6} m

Earth–Moon distance = 3.8×10^{5} km

Earth–Sun distance = 1.5×10^{8} km

1. You may sometimes find it difficult to get up from the sofa after watching a TV programme. Assuming the force of gravity acts between the centre of your body and the centre of the sofa, estimate the attraction between you and your sofa.

2. Calculate the size of the gravitational pull of a sphere of mass 10 kg on a mass 2.0 kg when their centres are 200 mm apart.

   What is the force of the 2.0 kg mass on the 10 kg mass?

3. At what distance apart would two equal masses of 150 kg need to be placed for the force between them to be 2.0×10^{-5} N?

4. Calculate the gravitational pull of the Earth on each of the following bodies:

   the Moon;

satellite A with mass 100 kg at a distance from the Earth's centre 4.2×10^7 m;

and satellite B mass 80 kg at a distance from the Earth's centre 8.0×10^6 m.

5. Show that the unit for G, the universal gravitational constant, can be expressed as $m^3 s^{-2} kg^{-1}$.

6. Calculate the weight of an astronaut whose mass (including spacesuit) is 72 kg on the Moon?

What is the astronaut's weight on Earth?

Comment on the difference.

7. Show that pull of the Sun on the Moon is about 2.2 times larger than the pull of the Earth on the Moon.

8. Why then does the Moon orbit the Earth?

9. The American space agency, NASA, plans to send a manned mission to Mars later this century. Mars has a mass 6.42×10^{23} kg and a radius 3.38×10^{6} m. G = 6.67×10^{11} N m^2 kg^{-2}

(a) The mass of a typical astronaut plus spacesuit is 80 kg. What would be the gravitational force acting on such an astronaut standing on the surface of Mars?

(b) State whether an astronaut on Mars would feel lighter or heavier than on Earth.

Answers and worked solutions

1. For the values estimated in the answers:

$$F = \frac{GMm}{r^2} = \frac{(6.67\times10^{-11}\ \mathrm{N\,kg^{-2}\,m^2})\times 60\ \mathrm{kg}\times 100\ \mathrm{kg}}{(0.5\ \mathrm{m})^2} = 1.6\times10^{-6}\ \mathrm{N}.$$

2. Pull on the 2.0 kg mass

$$F = \frac{GMm}{r^2} = \frac{(6.67\times10^{-11}\ \mathrm{N\,kg^{-2}\,m^2})\times 10\ \mathrm{kg}\times 2.0\ \mathrm{kg}}{(0.200\ \mathrm{m})^2} = 3.3\times10^{-9}\ \mathrm{N}.$$

The pull on the 10 kg mass will be equal but opposite in direction.

3.

$$r = \sqrt{\frac{Gm^2}{F}} = m\sqrt{\frac{G}{F}} = 150\ \mathrm{kg}\ \sqrt{\frac{6.67\times10^{-11}\ \mathrm{N\,kg^{-2}\,m^2}}{2.0\times10^{-5}\ \mathrm{N}}} = 0.27\ \mathrm{m}.$$

4.

$$F = \frac{GMm}{r^2} = \frac{(6.67\times10^{-11}\ \mathrm{N\,kg^{-2}\,m^2})\times(5.97\times10^{24}\ \mathrm{kg})\times(7.34\times10^{22}\ \mathrm{kg})}{(3.8\times10^{8}\ \mathrm{m})^2} = 2.0\times10^{20}\ \mathrm{N}.$$

$$F = \frac{GMm}{r^2} = \frac{(6.67\times10^{-11}\ \mathrm{N\,kg^{-2}\,m^2})\times(5.97\times10^{24}\ \mathrm{kg})\times 100\ \mathrm{kg}}{(4.2\times10^{7}\ \mathrm{m})^2} = 23\ \mathrm{N}.$$

$$F = \frac{GMm}{r^2} = \frac{(6.67\times10^{-11}\ \mathrm{N\,kg^{-2}\,m^2})\times(5.97\times10^{24}\ \mathrm{kg})\times 80\ \mathrm{kg}}{(8.0\times10^{6}\ \mathrm{m})^2} = 5.0\times10^{2}\ \mathrm{N}.$$

5. $\mathrm{N\,kg^{-2}\,m^2} = \mathrm{kg\,m\,s^{-2}\,kg^{-2}\,m^2} = \mathrm{m^3\,s^{-2}\,kg^{-1}}.$

6. Moon

$$F = \frac{GMm}{r^2} = \frac{(6.67\times10^{-11}\ \mathrm{N\,kg^{-2}\,m^2})\times 72\ \mathrm{kg}\times(7.34\times10^{22}\ \mathrm{kg})}{(1.64\times10^{6}\ \mathrm{m})^2} = 1.3\times10^{2}\ \mathrm{N}.$$

Earth

$$F = \frac{GMm}{r^2} = \frac{(6.67 \times 10^{-11}\ \mathrm{N\,kg^{-2}\,m^2}) \times 72\ \mathrm{kg} \times (5.97 \times 10^{24}\ \mathrm{kg})}{\left(6.37 \times 10^{6}\ \mathrm{m}\right)^2} = 7.1 \times 10^{2}\ \mathrm{N}.$$

7. Sun–Moon

$$F = \frac{GMm}{r^2} = \frac{(6.67 \times 10^{-11}\ \mathrm{N\,kg^{-2}\,m^2}) \times (2.0 \times 10^{30}\ \mathrm{kg}) \times (7.34 \times 10^{22}\ \mathrm{kg})}{\left(1.5 \times 10^{11}\ \mathrm{m}\right)^2} = 4.4 \times 10^{20}\ \mathrm{N}.$$

Earth–Moon

$$F = \frac{GMm}{r^2} = \frac{(6.67 \times 10^{-11}\ \mathrm{N\,kg^{-2}\,m^2}) \times (5.97 \times 10^{24}\ \mathrm{kg}) \times (7.34 \times 10^{22}\ \mathrm{kg})}{\left(3.8 \times 10^{8}\ \mathrm{m}\right)^2} = 2.0 \times 10^{20}\ \mathrm{N}.$$

$$\text{ratio of attractions} = \frac{4.4 \times 10^{20}\ \mathrm{N}}{2.0 \times 10^{20}\ \mathrm{N}} = 2.2$$

8. The Moon does of course orbit the Sun, as part of the Earth–Moon system. You can think of the Moon's orbit of the Earth as superimposed on its orbit of the Sun.

9 $F = (Gm_{astrom} \times M_{mars}) / r^2$

$F = (6.67 \times 10^{11}\ \mathrm{N\ m^2\ kg^{-2}}) \times 80\ \mathrm{kg} \times (6.42 \times 10^{23}\ \mathrm{kg}) / (3.38 \times 10^{6}\ \mathrm{m})^2 = 300\ \mathrm{N}$

(b) Would feel lighter.

External references

Questions 1- 8: This is taken from Advancing Physics Chapter 11, 80W

Question 9: This is taken from Salters Horners Advanced Physics, section STA, additional sheet 8 and 9